A simpler system of dimensions and units: part 2

Francis F. Pitard

In Part 1 (*TOS forum* 1/2) it was demonstrated that **time** and **mass** are relative concepts originating in the human imagination and it was postulated that they do not necessarily require units of their own. This constitutes but the tip of the iceberg, however, intended to furnish a simple start for the reader. Here the enquiry into a simpler system of dimensions and units continues deeper this time aiming at showing how worrisome the paradigm behind contemporary science is.

Electromagnetic quantities: eliminating the necessity of permeability and permittivity units of their own

The electrostatic dimensions of electric charge or quantity of electricity, Q, in the MLT system are:

$$[Q]=\frac{L^{1/2}\cdot M^{1/2}\cdot k^{1/2}}{T} \qquad (1)$$

or

$$[Q]=\left[\frac{\sqrt{L\cdot M\cdot k}}{T}\right] \qquad (2)$$

It is necessary to add electric permittivity, k to the dimensional statement. Magnetic statement of the dimensions of Q requires the use of magnetic permeability, u, instead of k.

In the magnetic system,

$$[Q]=\left[\sqrt{\frac{L\cdot M}{u}}\right] \qquad (3)$$

It is well established that the product $k\cdot u$ is numerically equal to $1/c^2$, where c is the velocity of light, so that:

$$[k\cdot u]=\left[\frac{1}{c^2}\right]=\left[\frac{T^2}{L^2}\right] \qquad (4)$$

In solving the problem of the height of liquid beads, earlier, we showed how to transfer from a MLT system to a $LP\rho$ system. The reason for this exercise may now appear clear. Changing from $MLTk\cdot u$ to $LP\rho$ enables us to eliminate k and u from the dimensional formula for the quantity of electricity and all electromagnetic quantities!

This is huge progress; again it becomes obvious time and mass do not need units of their own as they are relative concepts depending on other far more fundamental factors. It was also an easy way to demonstrate the relativity of time and mass.

Conversion of time, T, to the $LP\rho$ system gives

$$\left[T^2\right]=\left[\frac{L^2\cdot\rho}{P}\right],$$

therefore:

$$\left[\frac{1}{c^2}\right]=\left[\frac{T^2}{L^2}\right]=\left[\frac{L^2\cdot\rho}{P\cdot L^2}\right]=[k\cdot u]=\left[\frac{\rho}{P}\right] \qquad (5)$$

A note on the value of $k\cdot u$ is in order. Long before the relation $k\cdot u=1/c^2$ was established theoretically, it was observed to be true to the limits of measurements of the three quantities. It is also true that one may assign any dimension and any value to either k or u, provided the relation $k\cdot u=1/c^2$ is numerically and dimensionally satisfied. There should, therefore, be no objection not only to

$$[k\cdot u]=\left[\frac{T^2}{L^2}\right]=\left[\frac{\rho}{P}\right]\text{, but also to } [k]=\left[\frac{1}{P}\right]$$

and $[u]=[\rho]$.

We may now convert electromagnetic quantities from $MLTk\cdot u$ to LPD using:

$$[M]=\left[\rho\cdot L^3\right],\ \left[T^2\right]=\left[\frac{\rho\cdot L^2}{P}\right],$$

$$[k]=\left[\frac{1}{P}\right],\ [u]=[\rho]$$

Then, the quantity of electricity, Q is:

$$[Q]=\left[\frac{L^{\frac{3}{2}}\cdot M^{\frac{1}{2}}\cdot k^{\frac{1}{2}}}{T}\right]$$
$$=\left[L^{\frac{3}{2}}\cdot\rho^{\frac{1}{2}}\cdot L^{\frac{3}{2}}\cdot P^{\frac{-1}{2}}\cdot\rho^{\frac{-1}{2}}\cdot L^{-1}\cdot P^{\frac{1}{2}}\right]=\left[L^2\right] \qquad (6)$$

or

$$[Q]=\left[L^{\frac{1}{2}}\cdot M^{\frac{1}{2}}\cdot u^{\frac{-1}{2}}\right]=\left[L^{\frac{1}{2}}\cdot\rho^{\frac{1}{2}}\cdot L^{\frac{3}{2}}\cdot\rho^{\frac{-1}{2}}\right]=\left[L^2\right] \qquad (7)$$

The Potential Difference, E is:

$$[E]=\left[\frac{L^{\frac{1}{2}}\cdot M^{\frac{1}{2}}}{\sqrt{T\cdot k}}\right]$$
$$=\left[L^{\frac{1}{2}}\cdot\rho^{\frac{1}{2}}\cdot L^{\frac{3}{2}}\cdot P^{\frac{1}{2}}\cdot\rho^{\frac{-1}{2}}\cdot L^{-1}\cdot P^{\frac{1}{2}}\right]=[P\cdot L] \qquad (8)$$

or

$$[E]=\left[L^{\frac{3}{2}}\cdot M^{\frac{1}{2}}\cdot T^{-2}\cdot u^{\frac{1}{2}}\right]$$
$$=\left[L^{\frac{3}{2}}\cdot\rho^{\frac{1}{2}}\cdot L^{\frac{3}{2}}\cdot P\cdot\rho^{-1}\cdot L^{-2}\cdot\rho^{\frac{1}{2}}\right]=[P\cdot L] \qquad (9)$$

The product $Q\cdot E$ should, of course, have the dimensions of energy:

$$[Q\cdot E]=\left[P\cdot L^3\right]=\left[L^{-1}\cdot M\cdot T^{-2}\cdot L^3\right]=\left[\frac{M\cdot L^2}{T^2}\right] \qquad (10)$$

The $LP\rho$ system may be improved by noting that the dimensions of velocity are:

$$\left[\frac{L}{T}\right]=\left[\frac{P^{1/2}}{\rho^{1/2}}\right] \qquad (11)$$

and introducing a dimensional quantity, C, to replace, for convenience only, the otherwise clumsy L/T expression for velocity. We then have a $LP\rho C$ system. Some $LP\rho C$ dimensions of electromagnetic quantities follow.

Magnetic Field Strength, H:

$$[H]=[C] \qquad (12)$$

Electric Current, I:

$$[I]=[L\cdot C] \qquad (13)$$

Electric Resistance, R:

$$[R]=[\rho \cdot C] \tag{14}$$

Electric Inductance, $(E \cdot T)/I$:

$$\left[\frac{E \cdot T}{I}\right]=[\rho \cdot L]=[R \cdot T] \tag{15}$$

Magnetic Moment:

$$[L^3 \cdot C \cdot \rho] \tag{16}$$

Electric Moment:

$$[Q \cdot L]=[L^3] \tag{17}$$

The $LP\rho C$ system eliminates fractional exponents in dimensional expressions, provides a clear perception of the meaning of k and u, and simplifies dimensional operations in the "electromagnetic" system.

Solving an electromagnetic dimensional problem using both the $MLTk \cdot u$ and $LP\rho C$ systems is instructive.

Problem: Find the magnetic field strength at distance, d, from a magnet of length much less than this distance and with a magnetic moment, m. Table 2 shows the needed characteristics.

$$H=f(m,d,u)=c \cdot m^x \cdot d^y \cdot u^z \tag{18}$$

In *MLTu* terms,

$$\left[\frac{\sqrt{M}}{\sqrt{L \cdot T^2 \cdot u}}\right]=\left[\frac{\sqrt{L^5 \cdot M \cdot u}}{T}\right]^x \cdot [L]^y \cdot [u]^z \tag{19}$$

$$\frac{1}{2}=\frac{x}{2},\ -\frac{1}{2}=\frac{5x}{2}+y,\quad -1=-x,\ -\frac{1}{2}=z+\frac{x}{2} \tag{20}$$

In *LPDC* terms,

$$[C]=[L^3 \cdot \rho \cdot C]^x \cdot [L]^y \cdot [\rho]^z \tag{21}$$

$$1=x,\ 0=3x+y,\ 0=x+z \tag{22}$$

In either case,

$$x=1,\ y=-3,\ z=-1 \tag{23}$$

$$H=c\left[\frac{m}{u \cdot d^3}\right] \tag{24}$$

The constant c must be determined otherwise. If p is on the magnetic axis, $c=2$; if on the magnet's equatorial plane, $c=1$.

Evidently, the $LP\rho C$ approach is simpler. More importantly, **it removes the barrier that has existed between dynamical and electromagnetic units**. The reason for this is that the "dimensionally independent base units" of the *SI* system **are not dimensionally independent** as falsely claimed**.** Conversion to the $LP\rho C$ system demonstrates this. Note that *LMTu* yields four equations for only three unknowns.

Table 1. Comparison between *LMTku* and $LP\rho C$.

Physical quantity	Symbol	Dimensions LMTku	Dimensions LPDC
Magnetic field at p	H	$\left[\frac{\sqrt{M}}{\sqrt{L \cdot T^2 \cdot u}}\right]$	$[C]$
Magnetic moment of magnet	m	$\frac{\sqrt{L^5 \cdot M \cdot u}}{T}$	$[L^3 \cdot \rho \cdot C]$
Distance from magnet	d	$[L]$	$[L]$
Magnetic permeability	u	$[u]$	$[\rho]$

Thermal quantities

There seems to have been no serious attempt to weld thermal and mechanical dynamics into a single discipline. In the *MLT* part of the *SI* system, there is no mention of temperature. Thermodynamics makes no mention of time; its reasoning and the equations that express it use pressure, volume and temperature, P, V, T. The *SI* system, for obvious reasons, uses the symbol K for thermodynamic temperature, and this symbol is used here to replace the thermodynamic T, and also all kinetic symbols that have been used to represent temperature. The thermodynamic volume, V is rejected in favour of L^3, or $[V]=[L^3]$. As mentioned at the very beginning of this series, **the first step toward unification is a common language**! The discovery that $[k]=[1/P]$ and $[u]=[\rho]$ **has united electromagnetic and dynamics**. There must be a way to include thermodynamics, thermionics and kinetics in the scheme.

Incorporation of thermal quantities in the existing scheme will not be easy. Fundamental adjustments will have to be made; basic opinions and beliefs must be altered, if not completely overturned. In this attempt, we intuitively, or otherwise, guess that the proper dimensions of temperature are

$$[K]=[L^3 \cdot C^2]=\left[\frac{L^3 \cdot P}{\rho}\right]$$

in the $LP\rho C$ system, and therefore

$$\left[\frac{L^5}{T^2}\right]$$

in the *LMT* system.

Before dealing directly with thermal problems, it is well to recall the *SI* (2011–2012) definitions of the units in which the "**seven dimensionally independent quantities**" are measured.

Metre: the path length travelled by light in vacuum during a time interval of 1/299,792,458 of a second.

Kilogram: The mass of the international kilogram prototype.

Second: the duration of 9,192,631,770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.

Ampere: The constant current which, if maintained in two straight parallel conductors of negligible cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force of 2×10^{-7} Newton per metre length.

Kelvin: The fraction 1/273.16 of the thermodynamic temperature at the triple point of water.

Mole: the amount of substance in a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon 12.

Candela: the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} Hz and that has a radiant intensity in that direction of 1/683 W per steradian.

The symbols for these units are: m, kg, s, A, K, mol, cd, respectively.

These rather erudite definitions were designed to correct difficulties with original definitions of these quantities arising from the ever-increasing precision and accuracy of measurements. This does not remove the "King Henry's Thumb" nature of the original definitions, on which those written above are based. To unite the dynamic, electromagnetic and thermodynamic systems will require careful consideration of all these original definitions.

This will require revision of some favourite axioms.

The *SI* unitary system is based on properties of our earth. This may be better than a system based on King Henry's dimensions, but it is still arbitrary and hardly more fundamental!

There is a clear relation between kilogram, metre and second, and even a vague relation between these and temperature. The metre is a fraction of the earth's circumference; the kilogram is a mass (originally the weight) of a cubic decimetre of water, and thus is related to the kilometre and the earth's mass through the gravitational constant. The second is 1 / 86,000 of a solar day, and hence is related to the earth–sun distance in kilometres. The degree Kelvin is 1 / 100 of the difference in temperature of boiling and freezing water at 0.760 m of mercury of ambient pressure.

These units were adequate and appropriated when the earth was the centre of the Universe; they are no longer appropriate. Dressing them up by improving their definitions does not help, only more sweeping changes will.

To establish a new unitary system requires selection of more appropriate units of length, mass, time, electric current, temperature, amount of substance and luminous intensity. On the way, it may be demonstrated that these are not all dimensionally independent, as has been demonstrated for electric charge.

As a first step, we may examine the concept of length or distance, and area, volume and content, with the intent of selecting a new unit. **The idea of using a well-established wave length is appealing, and adds the possibility of finding a new unit of time during the same exercise.**

The dimensions of length

Length, L, is a vector quantity. It is necessary to distinguish dimensionally among $[L_x]$, $[L_y]$, $[L_z]$ and possibly $[L_t]$, where x, y, z refer to three spatial directions and t refers to time. Problems in dimensional analysis often require that the vector character of length be taken into account. A simple example may illustrate the concept.

Problem: Find the rate of fall of a small sphere through a viscous fluid. In a classic experiment, Millikan made use of Stokes' solution of this problem to measure the electron charge. Table 2 lists the relevant parameters, and

Table 2. Examples of physical quantities with their dimensions.

Physical quantity	Symbol	Dimensions
Velocity of sphere	v	$\left[\frac{L_z}{T}\right]$
Density of sphere	d	$\left[\frac{M}{L_x \cdot L_y \cdot L_z}\right]$
Diameter of sphere	r	$\left[L_x^{1/2} \cdot L_y^{1/2}\right]$
Density of liquid	ρ	$\left[\frac{M}{L_x \cdot L_y \cdot L_z}\right]$
Viscosity of liquid	n	$\left[\frac{M}{L_z \cdot T}\right]$
Acceleration of gravity	g	$\left[\frac{L_z}{T^2}\right]$

$$v=f(d^x,r^y,\rho^z,n^w,g^v) \tag{25}$$

In this case, it is not necessary to substitute $\rho \cdot L^3$ for M or L / C for T, the result is the same if this is done. The vector lengths are necessary for the solution, which is

$$v=\frac{(a_{\text{constant}})\cdot\left[r^2\cdot g\cdot(d-\rho)\right]}{n} \tag{26}$$

H.E. Huntley (reference 24 in our original textbook) gives a full explanation. Vector lengths are often necessary when angles are involved. The dimensions of an angle are not simply $[L / L]$, but may be $[L_X / L_Y]$ or, in the case of light velocity, not $[L / T]$ or $[\sqrt{P / \rho}]$, but $[L_x / L_t]$.

Candidates for the role of fundamental unit of length are the classic electron radius, r_e; the first Bohr radius, a_0; the Dirac Compton wave length, λ_c; and R_∞, the Rydberg constant:

$$r_e=\alpha^2\cdot a_0=2.81794092(38)\cdot 10^{-15}\,\text{m} \tag{27}$$

$$a_0=\frac{\alpha}{4\pi\cdot R\infty}=0.52917720859(36)\cdot 10^{-10} \tag{28}$$

$$\lambda_C=\frac{h}{m_e\cdot c}=2.4263102175(33)\cdot 10^{-12}\,\text{m} \tag{29}$$

$$R_\infty=\frac{m_e\cdot c\cdot\alpha^2}{2h}=10973731.568527(73)\,\text{m}^{-1} \tag{30}$$

With
fine structure constant:

$$\alpha=7.2973525376(50)\cdot 10^{-3} \tag{31}$$

electron mass:

$$m_e=9.10938215(45)\cdot 10^{-31}\,\text{kg} \tag{32}$$

light velocity:

$$c = 299792458 \text{ (exact) m s}^{-1} \tag{33}$$

Planck's constant:

$$\text{h}=6.62606896(33)\cdot 10^{-34}\,\text{J s}^{-1} \tag{34}$$

Numbers in parentheses refers to significant figures that are still uncertain.

Since the dimensions of light velocity are $[L / T]$, the logical unit of velocity is $1NU[L / T]$, of length is $1NU[L]$ and of time is $1NU[L / C] = 1NU[T]$. Choosing the conversion factors $[L]$ and $[T]$ from *NU* to *SI*, or *cgs,* or still another system, depends on the choice of factors from the above listed *SI* values. The two most precisely known values are those for R_∞ and c, but until the *NU* value for R_∞ is determined, it is best to begin with a_0 and c. Benefiting from arguments not presented here (see original book), the best value for the conversion factor $[L]$ is $(2.3^2\cdot\pi^2\cdot\alpha^2\cdot a_0)$ then:

$$1NU[L]=1\cdot(5.006151\ldots10^{-13}) \tag{35}$$

$$SI[L]=5.006148\ldots10^{-13}\,\text{m} \tag{36}$$

The conversion factor $[T]$ for time follows at once from the velocity of light, $c = 299792458\,\text{m s}^{-1}$.

$$1NU[T]=1NU\left[\frac{L}{c}\right]=1.\left[\frac{L}{c}\right] \tag{37}$$

$$SI[L]=1.669872\ldots10^{-21}\,\text{s} \tag{38}$$

In what follows, values for these conversion factors are carried to seven significant figures, because nearly all are valid to that level of precision, with eventual later establishment of the individual precisions.

The dimensions of mass

The difference between mass and weight is well recognised, but the *SI* unit of mass is the kilogram, a unit of weight on the earth's surface. There is a relation between volume density and mass.

$$\rho = \left[\frac{M}{L^3}\right] \text{ or } [M] = \left[\rho \cdot L^3\right] \qquad [39]$$

A distinction is necessary between mass as a quantity of matter, the gravitational mass, M_g, and inertial mass, M_i. Numerous repetitions of Eötvos' classic experiment with various modifications of a double-armed torsion balance have established that, in any unitary system, the values of M_i and M_g are identical. Gravitational and inertial mass are different concepts, related by a conversion factor, 1.00000…

The several Eötvos experiments have established this factor to a precision of at least one part in 10^9, and there is almost no doubt that $M_i / M_g = 1$. Inertial mass differs from gravitational mass in that it is a vectorial quantity. In the *LPρ* system, mass has dimensions $[M] = [\rho \cdot L^3] = [\rho \cdot L_x \cdot L_y \cdot L_z]$. The inertial mass of a projectile travelling in the *x* direction has dimensions $[\rho \cdot L_x \cdot L_y \cdot L_z]$.

The above discussion of the dimensions of length and mass are to be viewed as preliminary to a welding of all dynamic, electromagnetic and thermal quantities into a new, single unitary and dimensional system.

The techniques of dimensional analysis have been used above to show the validity of several arguments involving electromagnetic and dynamics. In Part 3 we will apply these techniques to thermal quantities. At this stage, keep in mind that visionary statement from Charles O. Ingamells:[1]

"If someone, somewhere, on some enchanted evening long ago, had decided, intuitively or otherwise, that the proper dimensions of electric charge are $[L^2]$, Physics today would be very different."

Slowly, but surely, for the reader implications may perhaps start to emerge far away on the horizon: are particles the way we think they are? Or, could it be that we got it wrong, and that they actually are something different, far more subtle to imagine? Our enquiry continues…

References

1. C.O. Ingamells and F.F. Pitard, *The Possibilities of our Sub-quantic Identity—The Theory of Vacuoles and A Simpler System of Dimensions and Units. Essay on Advanced Nuclear Physics*. Published by Francis Pitard, 14800 Tejon Street, Broomfield, CO, USA (2012). ISBN 978-0-9850631-0-8

Francis Pitard, gold medal recipient from the World Conference on Sampling and Blending, combines his experience in nuclear chemistry, analytical chemistry, geochemistry, and statistical process control in consulting and auditing of many international companies and teaching short courses on sampling. He lives in Colorado, USA.